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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2013/Jan.2014

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. **(06 Marks)**

b. If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are direction cosines of two lines then prove that the angle between them is $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$. **(07 Marks)**

c. Find the equation of the plane through the interaction of the planes $2x + 3y - z = 5$ and $x - 2y - 3z = -8$, also perpendicular to the plane $x + y - z = 2$. **(07 Marks)**

2 a. Prove that the equation of the plane in the intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **(06 Marks)**

b. Find the equation of the plane through the points (1, -2, 2) (-3, 1, -2) and perpendicular to the plane $2x - y - z + 6 = 0$. **(07 Marks)**

c. Find the angle between the following lines:

$$\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-3}{2} \quad \text{and} \quad \frac{x+1}{2} = \frac{y-3}{-1} = \frac{z-1}{0}$$ **(07 Marks)**

3 a. Find the sine of the angle between $\bar{a} = 2\bar{i} - 2\bar{j} + \bar{k}$ and $\bar{b} = \bar{i} - 2\bar{j} + 2\bar{k}$. **(06 Marks)**

b. Find the value of λ if the vectors $\bar{a} = 4\bar{i} + 6\bar{j} + 2\bar{k}$, $\bar{b} = 3\bar{i} + 10\bar{j} + 5\bar{k}$ and $\bar{c} = -4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar. **(07 Marks)**

c. Prove the following:

i) $(3\bar{a} - 2\bar{b}) \times (4\bar{a} + 2\bar{b}) = 14(\bar{a} + \bar{b})$

ii) $(2\bar{a} + 3\bar{b}) \times (\bar{a} + 4\bar{b}) = 5(\bar{a} + \bar{b})$ **(07 Marks)**

4 a. A particle moves along the curve $\vec{r} = (t^3 - 4t)\bar{i} + (t^2 + 4t)\bar{j} + (8t^2 - 3t^3)\bar{k}$. Find the velocity and acceleration at $t = 1$ and also find their magnitude. **(06 Marks)**

b. Find the unit normal vector to the surface $xy^3z^2 = 4$ at the point (-1, -1, 2). **(07 Marks)**

c. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\bar{i} + \bar{j} + 2\bar{k}$ **(07 Marks)**

5 a. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$, where $\vec{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$. **(06 Marks)**

b. Prove that curl grad $\phi = 0$. **(07 Marks)**

c. Find the constants a, b, c such that the vector $\vec{F} = (x + y + az)\bar{i} + (x + cy + 2z)\bar{k} + (bx + 2y - z)\bar{j}$ is irrotational. **(07 Marks)**

6 Find the Laplace transform of the following:

a. $\sin 4t \cos 3t$

b. cos hat

c. $t\, e^{-t} \sin t$

d. $\dfrac{1 - \cos t}{t}$ **(20 Marks)**

7 Find the inverse Laplace transform of

a. $\log\left(\frac{s+1}{s-1}\right)$ **(06 Marks)**

b. $\frac{s+1}{s^2+2s+2}$ **(07 Marks)**

c. $\frac{s}{(s+1)(s+2)(s-3)}$ **(07 Marks)**

8 a. By applying Laplce transforms, solve the differential equation $\frac{d^5y}{dt^2}+5\frac{dy}{dt}+6y=5e^{2t}$ subjected to the conditions $y(0) = y'(0) = 0$. **(10 Marks)**

b. Solve the simultaneous equations $\frac{dx}{dt}+y=\sin t$, $\frac{dy}{dt}+x=\cos t$ using Laplace transforms. Given that $x = 1$, $y = 0$ when $t = 0$. **(10 Marks)**

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